Florian Wickelmaier
Forschungsmethoden und Mathematische Psychologie
Fachbereich Psychologie
Universität Tübingen

Statistik III/Advanced Statistics
SS 2026

Exercises

Exercise 22. Box (1950) describes an experiment studying the effect of two substances on the growth of rats. For this purpose, 27 rats were randomly assigned to three groups, one of which had the hormone thyroxine (txn) added to the drinking water, and another group thiouracil (tcl), which inhibits the formation of thyroxine. These two groups were compared to a control group (ctl) kept with normal drinking water for four weeks. Body weight (bw, in grams) was measured in each case at the beginning (0) and at weekly intervals $(1, 2, 3, 4)$. The data are contained in the file `box-weight.txt`.

Read the data into R and save it to a data frame `dat.w`. Each time point is represented by one column (`bw0`, ..., `bw4`). Often, this kind of data format (wide format) is to be converted into a format (long format) that includes only one response variable (`bw`) and a time variable (`week`) per row. The `reshape()` function is available in R for converting data between wide and long format.

```
> dat <- reshape(dat.w, direction = "long", idvar = "rat",
+                varying = paste0("bw", 0:4), v.names = "bw",
+                timevar = "week", times = 0:4) |>
+   sort_by(~ grp + rat)
> dat$rat <- factor(dat$rat)
```

Refer to the data frame `dat` for further analysis. For all statistical descisions, specify the null hypthothesis and set the Type-I error rate $\alpha = 0.05$.

a) For ML parameter estimation with the `lmer()` function specify a preliminary mixed model with group-specific linear time trends as well as individual intercepts.

b) Extend the preliminary model by including individual linear time trends and specify the call to the `lmer()` function for ML estimation of its parameters.

c) Which of these models is to be preferred when compared by a likelihood ratio test?

d) Does the mean linear rate of weight change differ across groups? Answer this question by comparing appropriately formulated restrictions of the preferred model.

e) For each rat, plot the observed weight as a function of time. Add the weight predicted by the preferred model (`lattice` package allowed).

f) Interpret the parameter estimates of the fixed and random effects of the preferred model in the context of the specific application.

Reference:

Box, G.E.P. (1950). Problems in the analysis of growth and wear curves. Biometrics, 6, 362-389.