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Exercises

Exercise 17. The marketing department of a cosmetics company is conducting an experiment to study the effect of alternative design variants of a product on the sales. Sales (in units per 1000 customers) are monitored in 24 randomly selected stores (either (1) drugstores or (2) perfumeries). The design variants are (1) conservative, (2) neutral and (3) modern. The following data on the sales are available:

	store	
design	drugstore (ref)	perfumery
conservative	72, 78, 74, 70	66, 72, 68, 63
neutral (ref)	71, 71, 66, 70	73, 69, 64, 70
modern	70, 67, 66, 68	77, 71, 70, 72

The data are analyzed using the general linear model

$$Y_{ijk} = \mu_0 + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ijk}, \quad \varepsilon_{ijk} \sim N(0, \sigma^2) \text{ i.i.d.},$$

where μ_0 represents the mean sales in the reference class (neutral, drugstore).

a) Estimate the parameters of the model. Interpret the estimates.

b) Specify the ANOVA hypotheses that may be investigated in principle.

c) Test the effect of product design on sales ($\alpha = 0.05$)?

d) Graphically display the results of the study. Plot the average sales as a funtion of the design variants separately for each type of store. Explain how an interaction would be visible in such a display.

e) Which marketing strategy would you recommend (which product design should be implemented in which store)?

Exercise 18. Reconsider the `orthography.txt` data to predict the cognitive abilities `CFT` and `WM` as response variables Y_1 and Y_2 in a three-way factorial design by the factors A gender, B school type and C grade level.

Y_1	`CFT`	Culture Fair Intelligence Test (Subtest 1)
Y_2	`WM`	Phonological working memory performance (subtest of VLMT)
A	`sex`	Gender (dummy coded: 0 male, 1 female)
B	`school`	School type (dummy coded: 0 Hauptschule HS, 1 Realschule RS)
C	`class`	Grade level (dummy coded: 0 5th grade, 1 6th grade)

The following contingency table shows that the design is unbalanced:

School type		HS		RS	
Grade level		5	6	5	6
Gender	male	11	9	11	6
	female	17	13	5	28

Let the effects of the factors A, B, C be denoted by α, β, γ and the interactions by the respective combinations. Answer the questions using R, set $\alpha = 0.05$ for statistical tests.

a) Which additional effects are to be included in a general linear model that satisfies the principle of marginality if it contains the $\alpha\beta$ and $\alpha\gamma$ interactions?

b) Which are the effects that are to be adjusted for in a statistical test of the $\beta\gamma$ interaction according to type II sums of squares?

c) Test the $\beta\gamma$ interaction on the cognitive abilities `CFT` and `WM` within a multivariate model using type II sums of squares (either without or with the `car` package).

d) Interpret the results using a graphical display of the means (`lattice` package allowed).