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Statistik III/Advanced Statistics
SS 2026

Exercises

Exercise 15. A psychiatric clinic collects information on the variables

Y	Gerneral clinical impression (score, the higher the better)
X_1	Dose of an antipsychotic (in mg)
X_2	Days since hospitalization

and the following values are observed for four patients:

i	Y_i	X_{i1}	X_{i2}
1	40	1	36
2	45	2	33
3	50	3	37
4	65	4	37

To predict the general clinical impression, a regression model of the form

$$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \varepsilon_i \quad (i = 1, \ldots, 4)$$

is set up with $\varepsilon_i \sim N(0, \sigma^2)$ i.i.d. (independent and identically distributed). Perform all calculations by hand and using R.

a) Compute estimated values for $\boldsymbol{\beta} = (\beta_0, \beta_1, \beta_2)'$ and σ^2 according to the ordinary least squares method. Note: we have

$$(\mathbf{X}'\mathbf{X})^{-1} = \frac{1}{166}\begin{pmatrix} 23369 & 393 & -680 \\ 393 & 43 & -14 \\ -680 & -14 & 20 \end{pmatrix}$$

and

$$(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}' = \frac{1}{166}\begin{pmatrix} -718 & 1715 & -612 & -219 \\ -68 & 17 & 4 & 47 \\ 26 & -48 & 18 & 4 \end{pmatrix}.$$

b) Test whether the predictors contribute at all to a prediction of the general clinical impression ($\alpha = 0.05$).

c) Test the hypothesis

$$H_0 : \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} = \begin{pmatrix} 8 \\ 0 \end{pmatrix}$$

($\alpha = 0.05$).

d) Test the hypothesis that the days since hospitalization do not have an influence on the general clinical impression ($\alpha = 0.05$).