Statistik III

Advanced Statistics

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Organization

- Lecture
  - Tuesday, 8-10 am, HS 24 (Kupferbau)
- Exercises and Tutorials
  - One exercise sheet per week
  - Four parallel tutorials per week (see website for time slots)
- Examination
  - In-person computer exam (R)
  - Bring your own laptop
- Course material
  - Link Lehrveranstaltungen on the homepage of AB Forschungsmethoden und Mathematische Psychologie
  - Access directly via URL `https://www.mathpsy.uni-tuebingen.de/pw/`
  - Login details (Username, Password)

Introduction

The lecture aims to provide methods suitable for application to

- the simultaneous observation of multiple variables (multivariate methods)
- the development and design of diagnostic tests
- evaluation of interventions

The methods

- build on procedures and methods considered in introductory statistics courses
- generalize these methods in various respects
- account for frequently occurring complex data structures

Multivariate Statistical Methods

- in the broad sense: all statistical methods considering dependencies of more than two variables (irrespective of their classification as dependent or independent variables)
- Examples
  - Two factor Analysis of Variance (ANOVA) (2 IV, 1 DV)
  - Multiple linear regression ($>$1 IV, 1 DV)
- in the strict sense: all statistical methods considering more than one dependent variable (DV)
- Examples
  - One-factor multivariate analysis of variance (MANOVA) (e.g. 1 IV, 2 DV)
  - Multivariate (multiple) linear regression ($\geq$1 IV, $>$1 DV)
  - But not two-factor ANOVA (2 IV, 1 DV)

Multivariate Statistical Methods

- Many psychological constructs call for a multivariate operationalization, e. g. intelligence, personality, ...
- Applying multiple univariate statistical tests implies the problem of Type I error inflation, and the effects elicited by combinations of variables are left out of consideration
- If confounding variables cannot be controlled by design, we would at least want to estimate their effects by statistical computations

Multivariate Statistical Methods

- Identification of spurious effects
- Optimal prediction of dependent variables (criterion variables)
- Data reduction and structural simplification
- Appropriate classification techniques (e. g. in differential and diagnostic psychology)
- Examination of complex multi-causal dependencies
- ...

Mixed-Effects Models

- Data having a hierarchical multi-level structure are commonly encountered

  "Once you know that hierarchies exist, you see them everywhere" (Kreft & de Leeuw, 1998, p. 1)

- A multi-level structure is present if data on one level of analysis are hierarchically nested in a second level, such as
  - data of individuals in groups

level 3	school	clinic
level 2	class	clinic department
level 1	student	patient

  - data with repeated measurements (multiple measurements per person, longitudinal data)

Mixed-Effects Models

- Methods of change measurement

  Psychotherapy is fundamentally about change. Regardless of treatment modality, duration, setting, or any of a host of other considerations, when we study psychotherapy, we are primarily concerned with whether our clients changed as a result of our interventions. Thus, as treatment researchers, we should strive for expertise in the appropriate methods to model change.

  Atkins, D. C. (2005). Using multilevel models to analyze couple and family treatment data: Basic and advanced issues. *Journal of Family Psychology*, *19*, 98–110.

Mixed-Effects Models

Characteristics of hierarchical data structures

- Individual observations on level 1 are not independent
  - Not adequately modeling structure-induced dependencies can lead to distorted/biased estimates of effects, relationships and variances, as well as to incorrect statistical test results
- Statistical models for such data are called multilevel models, mixed-effects models, random-effects models, variance-component models, hierarchical models, ...

Mixed-Effects Models

- Considering the impact of attributes on different levels

level 3	school	school size, infrastructure
level 2	class	teacher experience and competencies, class size
level 1	students	cognitive skills, socioeconomic status, migration background

Mixed-Effects Models

Relationships on different levels

- Marsh, Köller & Baumert (2001) study the impact of math achievements on the self-concept of students and find a "Big-Fish-Little-Pond-Effect"
  - Students perceive their abilities more positive when coming from a weaker class (big fish in little pond)
  - Students with the same level of ability perceive it more negatively when coming from a relatively strong class (little fish in big pond)

Mixed-Effects Models

Relationships differ across levels

- At class level we observe a negative relationship between achievements and self-concept, meaning that for constant achievements we find a negative effect of the average class performance on the self-concept
- Whereas at the level of the individual student, achievement and self-concept exhibit a positive correlation, meaning that better performing students show higher self-concept

Mixed-Effects Models

Traditional statistical methods are applied within the following strategies

- In disaggregation studies, features of the higher level (e. g. groups) are assigned to the objects at the lower level (e.g. persons) and the analysis is done on the lower level
- In aggregation studies variables are aggregated/combined/summarized for every higher level object (e. g. groups) and analysis is done at the higher level (e.g. group level)

Problematic assumptions

- Independence of individual observations
- Relationships between variables at the lower level are assumed to be identical across higher level groups

Generalized Mixed-Effects Models

- Hierarchical data structures are very common in psychological diagnostics
- Example: In PISA, persons as well as items show a hierarchical structure

  persons students in classes — schools — countries

  items various items based on a single table, or graph

- Problem
  - How to take these structure-induced dependencies into account in statistical modeling?

Generalized Mixed-Effects Models

Background

- Test construction on the basis of models from Item Response Theory (Psychometrics)
- Logistic regression for binary prediction tasks (Statistics II and present lecture)
- Linear mixed models to analyze hierarchical data (present lecture)

In combining these approaches we show

- The Rasch model (and its extensions) can be conceived as a generalized linear mixed model (GLMM)

Topics

- Basics of linear algebra
- Basic concepts of multivariate statistics
- Multivariate hypotheses tests
- Multiple regression
- General linear model (GLM)
  - Analysis of variance
- Logistic and categorical regression
- Principle component analysis (PCA)
- Models for hierarchical data
  - (Generalized) mixed-effects models (LMM, GLMM)
- Model comparison
- Bayesian statistics
- Replication crisis

References

Marsh, H., Köller, O., & Baumert, J. (2001). Reunification of East and West German school systems: Longitudinal multilevel modeling study of the Big-Fish-Little-Pond-Effect on academic self-concept. *American Educational Research Journal*, 38, 321-350.

Nezlek, J.B., Schröder-Abé, M., & Schütz, A. (2006). Mehrebenenanalysen in der psychologischen Forschung Vorteile und Möglichkeiten der Mehrebenenmodellierung mit Zufallskoeffizienten. *Psychologische Rundschau*, 57, 213-223.