FS Exercises 02 (10 pts)

Simulation, PIT, rejection sampling

Exercise 2.1 (3 pts)

A Monte Carlo classic: Write R code to estimate the value of π by simulating values in a square centered at the origin and with side lengths 2. Count the proportion of points falling in the inscribed circle and multiply by 4 (why?) to get your estimate.

Exercise 2.2 (3 pts)

(Inspired by Shravan Vasishth.) Consider the random variable X and its density

$$p(x) = \tfrac{1}{40}(2x + 3), \quad 0 < x < 5.$$

- Show that $p(x)$ is a proper density (is positive and integrates to one).
- Derive the cumulative distribution function $P(x) = \int_0^x p(u)du$.
- Calculate $P^{-1}(y)$ by inverting $P(x)$.
- Calculate $E(X) = \int_0^5 xp(x)dx$. Hint: $E(X) = 3.021$.
- In R: Use PIT to sample from $p(x)$. Plot the samples. Estimate $E(X)$ from the samples.

Exercise 2.3 (4 pts)

Write R code to implement a rejection algorithm to sample from the standard normal distribution $p(x)$. Use as $g(x)$ the Laplace density and $C = \frac{2e^{\frac{1}{2}}}{\sqrt{2\pi}}$. Make a histogram of the samples.

Exercise 2.4 (2 bonus pts)

Write R code to implement a rejection algorithm to sample from the density in Exercise 2.2. Re-estimate $E(X)$.

Exercise 2.5 (2 bonus pts)

Write R code to implement importance resampling to sample from the density in Exercise 2.2. Re-estimate $E(X)$.